Received: *February 8, 2001,* Accepted: *April 17, 2001*

Comments on the holographic picture of the Randall-Sundrum model

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Abstract: We discuss some issues about the holographic interpretation of the compact Randall-Sundrum model, which is conjectured to be dual to a 4d field theory with non-linearly realized conformal symmetry. We make several checks of this conjecture. In particular, we show that the radion couples conformally to a background 4d metric. We also discuss the interpretation of the Goldberger-Wise mechanism for stabilizing the radion. We consider situations where the electroweak breaking stabilizes the radion and we discuss the issue of natural conservation of flavor quantum numbers.

Keywords: Extra Large Dimensions, AdS-CFT Correspondance.

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Contents

1. Introduction

The observed hierarchy between the Planck and Fermi scales suggests that there may be a more fundamental theory replacing the Standard Model (SM) just above the latter scale. Within this theory the hierarchy of scales should arise as a natural dynamical fact. This is what happens in technicolor and supersymmetry. Recently it was realized [1, 2, 3] that extra space dimensions can offer a new view point on the hierarchy problem, where the Fermi scale is essentially the fundamental quantum gravity scale and the weakness of 4d-gravity is explained by a very large volume of compactification. Later Randall and Sundrum (RS) [4] have pointed out that a small and warped extra dimension can elegantly solve the hierarchy problem. The basic idea is that we live on a 3-brane which is deep inside a five dimensional gravitational field [4]. Because of this, all the dimensionful parameters that describe the SM are redshifted with respect to possibly similar branes located at other points. In the RS set up (called from now on RSI) the fifth dimension is an $S_1/\mathbb{Z}_2$ orbifold with locally $AdS5$ geometry and bordered by two 3-branes with equal and opposite tensions. The AdS warp factor is exponential in the fifth coordinate so that the energy scales on the negative tension brane are also exponentially redshifted. If we live on this brane then we can naturally understand the small ratio M_W/M_P as due to an exponential red shift. Another way to view this is that the zero mode corresponding to the 4d graviton is localized at the positive brane (Planck brane) and has but a small overlap with the negative brane (TeV brane).

In the RSI model the graviton KK modes become strongly coupled among themselves and to the TeV brane just a little bit over a TeV, as expected in a theory where the fundamental quantum gravity scale is $\sim$TeV. On the other hand, an observer doing local measurements on the Planck brane would not see strong coupling phenomena at energies below $\sim M_P = 10^{19}$ GeV. One way to see this is that the light KK graviton modes couple very weakly to the Planck brane. The fact that observers at different points enter the strong regime at different energies can be confusing. What is the genuine scale of quantum gravity in this model? One way to address this question is to forget for a moment the hierarchy problem and send the negative brane all the way to the AdS horizon. Now spacetime is $AdS5$ wit the boundary region truncated and replaced by the Planck brane. Even though the fifth dimension is non-compact there still is a 4d graviton bound to the Planck brane. This is the RSII model [5]. Based on the AdS/CFT correspondence [6] is has been pointed out in [7, 8] that the RSII can be considered just a dual description of a strongly coupled 4d CFT where 4d gravity has been weakly gauged. In the 5d picture the weak gauging of 4d gravity is due to the presence of a truncation (Planck brane) removing the boundary of $AdS5$. In the limit where the Planck brane is moved to the boundary, 4d gravity decouples and one gets back just a four dimensional CFT.

This holographic duality is also useful to improve our intuition on the RSI model and on the way it solves the hierarchy problem [9, 10, 11]. From the holographic view point one can interpret the fifth coordinate as the renormalization scale in the 4d theory. The fact that $AdS5$ does not continue all the way down to the horizon but rather abruptly ends at the TeV brane corresponds to breakdown of conformal invariance below a TeV. The 4d theory localized on this brane is naturally interpreted as the low energy end point of RG evolution, *i.e* the low energy description of a strongly coupled (quasi)-CFT. Similarly, the Planck brane determines the UV boundary conditions while the fields that live on it represent some hidden sector coupled to the CFT only via $1/M_P$ suppressed operators. One may expect that in a more fundamental (string theory) description of the RSI set up, the metric singularity at the Planck and TeV branes will be smoothly resolved [9, 10]. So in the end the holographic interpretation of RSI is a 4d strongly coupled theory which stays essentially conformally invariant all the way down to the weak scale, below which it is effectively described by the Standard Model. This view point has been emphasized in [11] with various arguments. In this letter we will elaborate a little more on the interpretation of the RSI, by showing that the graviton-radion system is conformally invariant. We will also discuss the stability of M_W/M_P from the 4d point of view, which is crucial to decide the relevance of this scenario to the hierarchy problem. We will discuss the

4d interpretation of the Golberger-Wise (GW) mechanism for stabilizing the extra-dimension while obtaining a natural hierarchy [12]. We will argue that from the 4d point of view the GW mechanism is pretty generic. Similar arguments are presented in [11]. Furthermore we will consider more general situations where the electroweak breaking itself stabilizes the radion. Thinking of RSI as just a 4d field theory, rather than being reductive, we believe helps putting more theoretical requirements and therefore making more predictions. We will illustrate this by considering the issue of natural conservation of flavor (and baryon and lepton) quantum numbers.

In section 3 and 4, there is overlap with [11]. Some of the material and the motivations for this paper indeed resulted from discussions with the authors of [11].

2. GW radius stabilization: 5d picture

We briefly establish our notation. Consider a bulk five-dimensional theory

$$\mathcal{L} = \int d^5x\sqrt{-g}(2M^3R - \Lambda_5)\,. \tag{2.1}$$

The metric for RSI is given by

$$ds^2 = \frac{L^2}{z^2}\left(dx_\mu dx^\mu + dz^2\right), \tag{2.2}$$

where $L = 1/k$ is the AdS radius and the orbifold extends from $z = z_0 = L$ (Planck brane) to $z = z_1$ (TeV brane). Using as fifth coordinate $y = L\ln(z/L)$ we recover the usual parameterization with an exponential warp factor. z is the natural coordinate to discuss holography: $1/z$ can be interpreted as the renormalization scale. So the presence of the Planck brane at $z = z_0$ specifies that the 4d theory has a UV cut-off $\mu_0 = 1/z_0$, while $\mu_1 = 1/z_1$ represents the IR cut-off, the Fermi scale. The four dimensional Planck scale is given by

$$M_P^2 = M^3L^3\left(\frac{1}{z_0^2} - \frac{1}{z_1^2}\right) = (ML)^3(\mu_0^2 - \mu_1^2)\,. \tag{2.3}$$

The last expression is suited for a 4d interpretation [8, 11]: $c = 4\pi^2(ML)^3$ represents the central charge of the CFT [14, 15] and M_P is determined by a quadratically divergent quantum correction. In the limit $\mu_0 \to \infty$ 4d gravity decouples [11] and one recovers a pure CFT as in the AdS/CFT correspondence.

In the original RS paper the fifth dimension is an exactly flat direction. This is achieved by a suitable tuning of the model parameters. The tensions of the Planck and TeV branes are respectively given by $T_0 = +24M^3k$ and $T_1 = -24M^3k$, while the bulk cosmological constant is $\Lambda_5 = -24M^3k^2$. One of the tunings ensures that the effective 4d cosmological constant vanishes, while the other is equivalent to require that the radius be a flat direction [12].[1] The latter requirement is however not needed and actually phenomenologically unacceptable. Moreover, in order to argue that the model solves the hierarchy problem, a dynamical mechanism that naturally selects $z_1 \gg z_0$ must be found. Goldberger and Wise have shown that simply a bulk scalar field ϕ can do the job. The action of such field is

$$\int d^4x dz \left\{\sqrt{g}[-(\partial\phi)^2 - m^2\phi^2] + \delta(z - z_0)\sqrt{g_0}L_0(\phi(z)) + \delta(z - z_1)\sqrt{g_1}L_1(\phi(z))\right\}, \tag{2.4}$$

where $L_{0,1}$ are terms localized at the boundaries. Assume that the dynamics of such boundary terms is such as to fix the values $\phi(z_0) = \tilde{v}_0$ and $\phi(z_1) = \tilde{v}_1$. In the vacuum the field ϕ will have a bulk profile satisfying the 5d Klein-Gordon equation and with $\tilde{v}_0$, $\tilde{v}_1$ boundary values. The solution is then given by

$$\phi = Az^{4+\epsilon} + Bz^{-\epsilon} \tag{2.5}$$

with $\epsilon = \sqrt{4 + m^2L^2} - 2 \simeq m^2L^2/4$ for a small mass. The boundary conditions fix

$$A = z_1^{-4-\epsilon}\frac{\tilde{v}_1 - \tilde{v}_0(z_0/z_1)^{\epsilon}}{1 - (z_0/z_1)^{4+2\epsilon}}, \qquad B = z_0^{\epsilon}\frac{\tilde{v}_0 - \tilde{v}_1(z_0/z_1)^{4+\epsilon}}{1 - (z_0/z_1)^{4+2\epsilon}} \tag{2.6}$$

and eq. (2.4) evaluated on the solution yields an effective potential for the moduli z_0 and z_1

$$\begin{aligned} V(z_0, z_1) &= \frac{1}{1 - (z_0/z_1)^{4+\epsilon}}\left[(4+\epsilon)z_1^{-4}\left(v_1 - v_0\left(\frac{z_0}{z_1}\right)^{\epsilon}\right)^2 + \epsilon z_0^{-4}\left(v_0 - v_1\left(\frac{z_0}{z_1}\right)^{4+\epsilon}\right)^2\right] \\ &= z_0^{-4}F\left(\frac{z_0}{z_1}\right), \end{aligned} \tag{2.7}$$

where $v_{0,1} = L^{3/2}\tilde{v}_{0,1}$ are the boundary vacuum expectation values (VEVs) in units of the AdS curvature. The minimum of F, if it exists, determines the size of the compact dimension. Notice that the value of F at this point, plays the role of an effective 4d cosmological constant. Stationarity in the conformal factor z_0 as well requires a vanishing F at the minimum. As usual this can be achieved by fine tuning extra contributions to the effective potential. For instance by changing the tensions of the branes with respect to the RS values $T_0 = -T_1 = 24M^3k$ one gets a correction

$$\delta V(z_0, z_1) = z_0^{-4}\delta T_0 + z_1^{-4}\delta T_1 \tag{2.8}$$

so it is enough to properly choose δT_0 to get a vanishing effective cosmological

[1] Basically this can also be seen as flatness of the two moduli corresponding to the brane positions z_0 and z_1. Notice however that only one combination, say z_0/z_1 corresponds to a 4d scalar. The other, say z_0, represents the conformal factor of the 4d metric.

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constant at the minimum of V. Since we are not interested in the 4d gravitational dynamics and in the cosmological constant we will freeze z_0 to a constant $= 1/\mu_0$ and keep $\mu = 1/z_1$ as our radion field. We are interested in a situation where a huge hierarchy $\langle\mu\rangle = \mu_1 \ll \mu_0$ arises. This can be naturally achieved if $|\epsilon|$ is somewhat smaller than 1. In the region $\mu \ll \mu_0$ we can expand eq. (2.7) as

$$V = \epsilon v_0^2 \mu_0^4 + \left[(4+2\epsilon)\mu^4 (v_1 - v_0(\mu/\mu_0)^\epsilon)^2 - \epsilon v_1^2 \mu^4\right] + \mathcal{O}(\mu^8/\mu_0^4)\,, \tag{2.9}$$

where in estimating the remainder we have assumed that $|\epsilon| \ll 1$. For $\epsilon > 0$ the above potential has a minimum around $\mu = \mu_0(v_1/v_0)^{1/\epsilon}$. The hierarchy $\langle\mu\rangle/\mu_0 \sim M_W/M_P = 10^{-17}$ can be naturally obtained for fundamental parameters not much smaller than one (ex. $v_1/v_0 \sim 1/10$ and $\epsilon \sim 1/20$). Notice that the hierarchy originates because the relevant part of the potential is of the form $\mu^4 P(\mu^\epsilon)$, i.e. a basically scale invariant function modulated by a slow evolution through the μ^ϵ terms. This is very much what happens in the Coleman-Weinberg mechanism [13], where a slow RG evolution of the scalar potential parameters can generate wildly different mass scales. As we will see below this is not surprising from the CFT point of view of GW.

Notice that in AdS one can also consider "tachyonic" scalars provided $m^2 > -4k^2$ without introducing instabilities [14].[2] So in eq. (2.9) we can also consider $\epsilon < 0$. One finds that for this case the minimum sits at $\mu = 0$. However, this does not mean that the $\epsilon < 0$ cannot be used. We can (should) always imagine that there are extra terms coming from the brane tensions as in eq. (2.8). It is easy to see that for a range of δT_1 there is a global minimum at finite μ also when $\epsilon < 0$.

The above calculation of the radion potential neglects the backreaction of the background metric on the scalar energy momentum density. The latter is proportional to $\tilde{v}_{0,1}^2 = \mathcal{O}(\tilde{v}^2)$. So, based on simple dimensional analysis, we expect our potential to be the leading result in an expansion in $\tilde{v}^2/M^3$. One can easily estimate the corrections to the radion (and massless graviton) effective action, by integrating out at tree level the KK modes around the GW approximate solution. It is easy to see that lowest order exchange of the massive gravitons yields a correction to the potential $\delta V \sim \tilde{v}^4/M^3$, which has the expected suppression with respect to the leading result, eq. (2.7). The leading corrections to the radion kinetic term come instead from the KK excitations of the scalar ϕ. Notice first of all that there is no approximate zero mode in this sector, the lowest excitation has a mass of order $1/z_1 = \langle\mu\rangle$, so this procedure makes sense. Now, since for spacetime independent moduli $z_{0,1}$ the scalar action is stationary around eq. (2.5), there can only be a kinetic mixing between μ and the KK modes. By integrating these out one gets a

[2] This also works in a slice of AdS by imposing Dirichlet boundary condition $\delta\phi = 0$ at the Planck brane. This is the case of the GW scalar. On the other hand, a scalar with $\partial_z\phi = 0$ at both boundaries is unstable for $m_\phi^2 < 0$.

leading correction $\sim \tilde{v}^2 L^3 (\partial\mu)^2$, which should be compared to the lowest order result $L_{kin} \sim M^3 L^3 (\partial\mu)^2$. In appendix A we will discuss in some more detail the radion mass generated by the GW mechanism. We will also comment on the particular set up [16] where the backreaction is automatically included.

In the next sections, using holography, we will give a purely 4d interpretation of the previous results.

3. The holographic interpretation of the RSI model

Using the standard rules of AdS/CFT, we interpret the fifth coordinate of AdS as an energy scale. The region between the two branes represents the energy regime $\mu_0 \gg E \gg \mu_1$ where the 4d theory is well approximated by a CFT. The Planck brane represents the UV cut-off. Its dynamics determines the boundary conditions for bulk fields: by holography these correspond to boundary conditions on the coefficients of deformations of the CFT. Moreover fields that are just localized on the Planck brane look like external moduli from the point of view of the CFT: these fields are very much like a hidden sector coupled to the CFT by $1/\mu_0$ suppressed operators[3]. Finally the TeV brane roughly describes the IR limit of the CFT, very much like the chiral lagrangian does in QCD.

The TeV brane abruptly ends AdS space, signalling breakdown of conformal invariance in the IR. The first question then is: what kind of breaking is this? Explicit (soft) or spontaneous? The first possibility is the same as saying that some relevant deformation is turned on in the CFT, eventually generating a mass gap. This scalar operator would have to be associated through AdS/CFT to some 5d scalar with negative mass^2. The minimal RSI model (though phenomenologically unacceptable) is perfectly consistent without any such scalar. This leaves spontaneous breaking as the only viable option. It is also intuitively clear that this is what happens. The position of the IR brane, which sets the mass scale of the model is determined by the expectation value of a dynamical radion field. All KK masses scale like $\mu_1 = \langle\mu\rangle$ while the coupling among them scales like $1/(c\mu_1)$, with $c \sim (ML)^3$. In minimal RSI this field is an exact modulus, so it is naturally interpreted as the Goldstone boson of broken dilatation invariance.

We can consider the RSI model as an idealized description of a CFT along an exactly flat direction parameterized by the radion. Situations in the AdS/CFT correspondence where the conformal invariance is spontaneously broken can be easily found. For example, the Coulomb branch of $N = 4$ SYM, described by D3-branes sitting at different points in ten dimensions. In this case, the adjoint scalars of $N = 4$ SYM have a VEV and the radion parameterizes the overall magnitude of the scalar

[3] This is not true if there are gauge fields in the bulk [19, 20, 11] under which the Planck brane fields are charged. In this case the coupling between the hidden sector and the visible TeV brane is only suppressed by $1/\ln(\mu_0/\mu_1)$.

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VEVs. This is a analogy that should be taken with some care. The RSI model is not supersymmetric and the flat direction is not necessarily associated with a Coulomb branch or a scalar VEV.

We can make several checks of the spontaneous breaking of conformal invariance by adapting the rules of AdS/CFT to our case. AdS/CFT allows to compute Green's functions of composite operators of the CFT. We can then check that the trace of the stress-energy tensor is unchanged by the presence of the IR brane. Moreover, the dilatation current two-point function has a pole, as dictated by Goldstone's theorem. In order to simplify things we can decouple 4d gravity by sending the UV brane all the way to the AdS boundary at $z = 0$. We now compute the Weyl anomaly of the dual quantum field theory following [15]. We can restrict our analysis to the UV region. An explicit breaking of conformal invariance would affect the trace of the stress-energy tensor and would be already visible in the UV. The rules of AdS/CFT state that the CFT partition function in presence of a gravitational background $g^{(0)}_{\mu\nu}(x)$ is given by the classical 5d action $S[g^{(0)}]$ for a solution

$$ds^2 = \frac{L^2}{z^2}\left(g_{\mu\nu}(x,z)dx^\mu dx^\nu + dz^2\right) \tag{3.1}$$

of the 5d Einstein's equations satisfying the boundary condition $\lim_{z\to 0} g_{\mu\nu}(x,z) = g^{(0)}_{\mu\nu}(x)$ plus the requirement that $g_{\mu\nu}(x,z)$ behaves well at the AdS horizon $z \to \infty$. In our case the latter condition will be replaced by the orbifold Israel junction conditions at the IR brane. In AdS/CFT, Weyl invariance shows up in the fact that two conformally related boundary metrics $g^{(0)}_{\mu\nu}(x)$ and $e^{2\sigma(x)}g^{(0)}_{\mu\nu}(x)$ determine the same solution, up to diffeomorphisms. This can be easily seen at the infinitesimal level by working around exact AdS. Consider the coordinate change

$$x^\mu = x'^\mu + f^\mu(x')A(z) \qquad z = z' - f(x')A_{z'}(z') \tag{3.2}$$

with $A_{z'} = dA/dz'$, $f^\mu = \eta^{\mu\nu}\partial_\nu f$ and with $A(z') \to -z'^2/2$ for $z' \to 0$ so that the boundary $z = 0$ is kept fixed. The AdS metric of eq. (2.2) is changed to

$$\begin{aligned} ds^2 = \frac{L^2}{z'^2}\Bigg\{&\left[\left(1+\frac{2f(x')A_z(z')}{z'}\right)\eta_{\mu\nu} + 2A(z')f_{\mu\nu}(x')\right]dx'^\mu dx'^\nu + \\ &+\left[1+2f(x')\partial_{z'}\left(\frac{A_{z'}}{z'}\right)\right]dz'^2\Bigg\}. \end{aligned} \tag{3.3}$$

Now by choosing exactly $A(z') = -z'^2/2$ the metric is again of the form in eq. (3.1) but the corresponding boundary value is now $g^{(0)}_{\mu\nu} = (1 + 2f(x'))\eta_{\mu\nu}$. The presence of the IR brane does not change things much: the only difference is that in the new coordinates the position of the IR brane is given by $z' \simeq z_1 + f(x)A_z(z_1)$. However, even though the IR brane is apparently "bent", the induced metric on it is still flat since the bulk geometry has not changed. The above argument can be easily extended to infinitesimal rescalings around arbitrary $g^{(0)}_{\mu\nu}(x)$.

By the above argument the partition function $S[g^{(0)}]$ in our case, like in standard AdS, should be invariant under Weyl rescalings. Things are however slightly more involved since $S[g^{(0)}]$ is divergent and must be regulated. The divergences are originated by the region $z \to 0$ where the AdS "volume" element grows very fast. The regularization can be done by restricting the integration to the region $z > \epsilon$, after which the action can be renormalized as $\epsilon \to 0$ by adding local counterterms covariant in $g^{(0)}$. In usual AdS/CFT it has been shown [15] that at the end of this procedure the renormalized action $S_R[g^{(0)}]$ is Weyl invariant up to an anomaly local and covariant in $g^{(0)}$. This is precisely what one expects for a CFT in a non trivial background. Now, it is easy to verify that the Weyl anomaly calculation of ref. [15] goes through unchanged in the presence of our infrared brane. That calculation is based on the expansion of the solution $g_{\mu\nu}(x,z)$ around $z=0$

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + z^2 g^{(2)}_{\mu\nu} + z^4 g^{(4)}_{\mu\nu} + z^4 \ln z h^{(4)}_{\mu\nu} + \cdots . \tag{3.4}$$

The point is that the anomaly depends only on $g^{(2)}$ and $\mathrm{Tr}\, g^{(0)-1} g^{(4)}$, which algebraically depend on $g^{(0)}$, and the boundary conditions at the IR brane (which start to matter at order z^4) do not affect these terms. We conclude that the RSI model in the limit where 4d gravity decouples is conformal.

The conformal symmetry is however non-linearly realized in RSI. This can formally be seen by the fact that under the conformal transformation $x \to \lambda x$, $z \to \lambda z$, the AdS isometry, the position of the IR brane is changed $z_1 \to \lambda z_1$. The physics is however unchanged, so that z_1 parameterizes a manifold of equivalent vacua. The associated Goldstone boson is the radion μ. This can more directly be seen by considering the effective lagrangian for radion and 4d gravity calculated in refs. [17, 18]

$$\mathcal{L} = \sqrt{g} M^3 L^3 \left\{ 2(\mu_0^2 - \mu^2) R(g) - 12 (\partial\mu)^2 \right\} . \tag{3.5}$$

When $\mu_0 \to \infty$ 4d gravity decouples and g is just a background probing our CFT (consistently, the induced metric on the Planck brane is exactly g). For this purpose notice that the μ dependent terms are formally Weyl invariant

$$\sqrt{g} M^3 L^3 \left\{ -2\mu^2 R(g) - 12(\partial\mu)^2 \right\} = -\sqrt{g} M^3 L^3 \mu^4 R(\mu^2 g) . \tag{3.6}$$

Moreover μ couples to the TeV brane only in the Weyl invariant combination $\mu^2 g_{\mu\nu}$. By integrating out the radion at tree level eq. (3.6) gives a contribution $S_{rad}[g]$ to the source action

$$S_{\mathrm{rad}}[g] = M^3 L^3 \int \sqrt{g} d^4x \left(-2\mu_1^2 R(g) - \frac{\mu_1^2}{3} R(g) \Box^{-1} R(g) \cdots \right) . \tag{3.7}$$

S_{rad} is invariant under Weyl transformations $g \to e^{2\sigma(x)} g$ with $\sigma(x)$ vanishing fast enough at infinity, but not under strictly rigid rescaling $g \to \lambda g$. The rigid dilatations cannot be smoothly obtained from the local ones because of surface terms in the variation of S_{rad}, a signal of spontaneous symmetry breaking. By varying S_{rad} twice with respect to $g_{\mu\nu}$ we obtain the radion contribution to the two-point function of $T_{\mu\nu}$. Eq. (3.7), being Weyl invariant, only contributes to the transverse-traceless part of the correlator:

$$\langle T_{\mu\nu}(x)T_{\rho\sigma}(0)\rangle = -\frac{2}{3}(ML)^3\mu_1^2 \prod^{(2)}_{\mu\nu\rho\sigma}\frac{1}{\Box}\delta^4(x)\,, \tag{3.8}$$

where $\pi_{\mu\nu} = \partial_\mu\partial_\nu - \eta_{\mu\nu}\partial^2$ is a projector over traceless tensors, and $\prod^{(2)}_{\mu\nu\rho\sigma} = 2\pi_{\mu\nu}\pi_{\rho\sigma} - 3(\pi_{\mu\rho}\pi_{\nu\sigma} + \pi_{\mu\sigma}\pi_{\nu\rho})$. In analogy with current algebra, the Goldstone boson creation amplitude is

$$\langle 0|T_{\mu\nu}|rad\rangle = \sqrt{\frac{8}{3}}(ML)^{3/2}\mu_1 p_\mu p_\nu \tag{3.9}$$

and $\sqrt{8/3}(ML)^{3/2}\mu_1$ is the radion decay constant. The same Goldstone pole appears in the correlator $\langle S_\mu(x)S_\nu(-x)\rangle$ of the dilatation current $S_\mu = x^\nu T_{\mu\nu}(x)$.

The same pole should appear in the energy momentum correlator computed using the rules of AdS/CFT. The two point function of a conserved stress-energy tensor has the general form [21]

$$\langle T_{\mu\nu}(x)T_{\rho\sigma}(0)\rangle = -\frac{1}{48\pi^4}\prod^{(2)}_{\mu\nu\rho\sigma}\left[\frac{c(x)}{x^4}\right] + \pi_{\mu\nu}\pi_{\rho\sigma}\left[\frac{f(x)}{x^4}\right], \tag{3.10}$$

For a conformal theory, $f = 0$ and $c(x)$ is constant and equal to the central charge $c = 4\pi^2(ML)^3$ of the theory. In a spontaneously broken theory, f still vanishes but $c(x)$ is non-trivial. From eq. (3.8), we expect that the Fourier transform of $c(x)/x^4$ has a pole $1/p^2$ for $p \to 0$. Let us compute this quantity using the rules of AdS/CFT. The computation of the boundary effective action is essentially like in ref. [14]. The horizon region has been replaced with the TeV brane, and the field behavior at the horizon replaced by the orbifold conditions at the TeV brane. The transverse-traceless part of the two-point function of $T_{\mu\nu}$ can be read from the two-point function of a minimally coupled scalar field, which is explicitly computed in appendix B. The results is

$$\mathcal{F}(p) = \int d^4p e^{ipx}\frac{c(x)}{x^4} = -2\pi^2 c\left(\log\frac{p}{2} - \frac{K_1(p/\mu)}{I_1(p/\mu)}\right) \tag{3.11}$$

which has indeed a pole $\mathcal{F}(k) \sim 16\pi^4\mu^2(ML)^3/p^2$ with the right coefficient.[4]

There is still an important point to be addressed. By the above discussion, corresponding to a radion value $\langle\mu\rangle = \mu_1$ there should be some CFT operators O_i of dimensions d_i getting VEVs $\langle O_i\rangle = c_i\mu_1^{d_i}$, with c_i constants. (This scaling in μ is due to the absence of explicit conformal breaking sources or, better, to conformal covariance.). The large euclidean momentum behavior of correlators from the point

[4]The factor 2 mismatch is trivially due to the doubling of the RS action on the orbifold with respect to the standard AdS/CFT computation where only one slice of AdS is present.

of view of the OPE gives information on the dimensions of the O_i. For the two point function at momentum p of an operator of dimension d we expect

$$\langle O_d O_d \rangle_{p^2} = \frac{b_0}{p^{4-2d}} + \sum_i b_i \frac{\langle O_i \rangle}{p^{4-2d+d_i}}. \tag{3.12}$$

Such correlators can be calculated using the holographic prescription. The example for a dimension four operator is discussed in Appendix B. The general result [11] is that at euclidean $p \gg \mu$ the deviation from conformality is suppressed like $\exp(-p/\mu)$ rather than being just power suppressed as naively expected from eq. (3.12). The 5d reason behind this wild suppression is that the local geometry between the two branes is *exactly* $AdS5$. The natural 4d interpretation of this result is that the operator that spontaneously breaks conformal invariance has formally infinite dimension. In AdS/CFT small curvature on the AdS side corresponds to large N and large 't Hooft parameter gN in the CFT. So it is conceivable that the operators involved in symmetry breaking have inverse dimensions $1/d$ which vanish at lowest order in the large N and large gN expansion.[5] In the AdS side finding the dimensions of these operators requires a smoothing of the jump singularity at IR brane by lifting the RS model to 10 dimensions and by accounting for string modes.

4. GW radius stabilization: holographic interpretation

We saw that conformal breaking in the vacuum is due to the vevs $\langle O_i \rangle = c_i \mu^{d_i}$ of one or more operators with a "large" dimension. In general, corresponding to this field configuration, there will be an effective potential. By conformal invariance this must be of the form $V_{\text{eff}} = a\mu^4$, with a constant. Of course in minimal RSI, $a = 0$ and all μ's have the same energy. However changing the TeV brane tension, like in eq. (2.8), corresponds to having $a = \delta T_1$. Then for $a > 0$ the TeV brane is pushed to the horizon and conformal invariance is unbroken in the vacuum, while for $a < 0$ the system is destabilized (the TeV brane falls towards the Planck one). So, only for $a = 0$ is conformal invariance truly spontaneously broken with the radion being the corresponding Golstone boson.

By the above discussion, in order to get both $\langle \mu \rangle \neq 0$ and no Goldstone mode, it is necessary to introduce some explicit source of conformal breaking. The simplest

[5] All the string states, which certainly populate the AdS background, correspond to operators with large dimension $\sim \sqrt{gN}$ [14] The operators O_i, corresponding to a set of supergravity modes with large mass, would certainly mix with string states in the IR. As a toy model, we could consider a distribution of D3-branes in type-IIB. A quasi-spherical distribution with only high multipoli momenta gives VEV only to the operators $O_k = \mathrm{Tr}(\phi_{\{i_1} \ldots \phi_{i_k\}}) -$ traces, $k \geq k_0$ and we can take k_0 arbitrarily large. A completely spherical distribution has all the multipoli VEVs $\langle O_k \rangle = 0$ but has $tr\phi_i\phi_i \neq 0$. It is well known that the trace $tr\phi_i\phi_i$ is the prototype of a stringy mode in the AdS/CFT correspondence for $N = 4$ SYM. It would be interesting to find an explicit type-IIB model which mimics all the relevant features of the RSI model.

thing to do is to add a perturbation

$$\delta L = \lambda \mathcal{O}\,, \tag{4.1}$$

where $\mathcal{O}$ is an operator of dimension $[\mathcal{O}] = 4 + \epsilon$. For $|\epsilon| \ll 1$ we may call the perturbation *almost marginal*. The coupling evolves with the renormalization scale Q as $\lambda(Q) = \lambda(\mu_0)(Q/\mu_0)^\epsilon$. By RG invariance the effective potential will now have the form

$$\begin{aligned} V_{\rm eff} &= \mu^4 P(\lambda(\mu)) = a\mu^4 + \mu^4 \sum_{n=1} a_n \lambda^n(\mu) \\ &= \mu^4 \left(a + a_1\lambda(\mu_0)(\mu/\mu_0)^\epsilon + a_2\lambda(\mu_0)^2(\mu/\mu_0)^{2\epsilon} + a_3\lambda(\mu_0)^3(\mu/\mu_0)^{3\epsilon} + \cdots\right). \end{aligned} \tag{4.2}$$

Notice that the finite part of eq. (2.9) is just a special case of this where $\lambda(\mu_0) = v_0$. The discussion of the previous section on the possibility of generating minima with $\mu/\mu_0 \ll 1$ applies. Of course we would need to be able to calculate the coefficients a_i in the CFT in order to say something on the vacuum.

What we have outlined in the previous paragraph is just the 4d picture of the GW mechanism. In the AdS/CFT dictionary a bulk scalar ϕ with mass m^2 corresponds to a scalar operator $\mathcal{O}$ of dimension $d = \sqrt{4 + m^2/k^2} + 2 = 4 + \epsilon$. The two independent solutions $z^{4+\epsilon}$ and $z^{-\epsilon}$ of $\Box\phi = 0$ in the AdS background are respectively associated to the VEV $\langle\mathcal{O}\rangle$ and to the source λ. More precisely, given the profile

$$\phi = Az^{4+\epsilon} + Bz^{-\epsilon} \tag{4.3}$$

AdS/CFT relates [14, 22]

$$A = \frac{\langle\mathcal{O}\rangle}{4 + 2\epsilon} \qquad B = \lim_{\mu_0\to\infty} \mu_0^{-\epsilon}\lambda(\mu_0)\,. \tag{4.4}$$

Confronting with eq. (2.6), we basically have that $\tilde{v}_0 = \lambda(\mu_0)$ is the UV deformation parameter. Moreover by taking the limit $\tilde{v}_0 = 0$ we have $\langle\mathcal{O}\rangle = (4+2\epsilon)\mu^{4+\epsilon}\tilde{v}_1$. So the two vevs v_0 and v_1 are respectively associated to explicit and spontaneous breaking of conformal invariance.

Eqs. (4.4) can be checked directly using the holographic potential in eqs. (2.7) and (2.9). In order to avoid spurious cut-off effects one should take the limit $z_0 \to 0$ with the scaling $v_0 = \bar{v}_0 z_0^{-\epsilon}$ and also renormalize eq. (2.7) by subtracting the μ independent $1/z_0^4$ divergent term. One gets

$$V_{\rm ren} = \left[(4 + 2\epsilon)\mu^4(v_1 - v_0(\mu/\mu_0)^\epsilon)^2 - \epsilon v_1^2\mu^4\right] \tag{4.5}$$

from which

$$\langle\mathcal{O}\rangle = -\mu_0^\epsilon \frac{\partial}{\partial\lambda(\mu_0)} V_{\rm eff} \equiv -z_0^{-\epsilon}\frac{\partial}{\partial v_0} V_{\rm ren} = 2(4 + 2\epsilon)(v_1 - \bar{v}_0\mu^\epsilon) \tag{4.6}$$

which is consistent with eqs. (2.6), and (4.4).[6]

[6] Once again, the factor 2 mismatch is due to the doubling of the RS action on the orbifold, with respect to ref. [22] where there is just one branch of AdS.

Finally one should not be puzzled by $V_{\rm ren}$ being only quadratic in the perturbation. This is because the GW scalar is a free field and correspondingly the connected Green's functions with more than two $\mathcal{O}$ legs vanish [14]. By considering a self-interacting ϕ we would get an effective potential of the general form (4.2). In appendix A we further discuss the radion potential and mass in the general case.

By considering its 4d picture we can better understand in which sense the RS model solves the hierarchy problem. Conformal symmetry as opposed to supersymmetry was invoked by Frampton and Vafa [23] as a principle to cure the destabilizing quadratic divergences of the SM. However, as we will now discuss, it does not work as well. It is certainly acceptable to elevate the conformal group to a fundamental symmetry. Then this symmetry has to be valid at all energy scales somewhat larger than the weak scale. However this does not help the hierarchy problem, which is one of separation of mass scales, i.e. the separation of the Fermi scale from the Planck or GUT scales. The presence of these other scales implies that the theory can only be conformal in a limited energy regime. This is to say that the conformal symmetry is not fundamental but dynamical, and we cannot use it as a principle to discard unwanted parameter choices. In general these perturbations will be there and in order for the model to be interesting they should not badly affect the hierarchy. Then the question whether a CFT can solve the hierarchy problem depends on the classification of its relevant deformations. If the theory admits deformations of dimension 2 or 3, which cannot be discarded by independent symmetry considerations, then the electroweak hierarchy is badly unstable. This is truly the case of the SM, which being weakly coupled is almost a CFT, and which admits a dimension 2 Higgs mass deformation. As shown in ref. [24] this is also what happens in the model of ref. [23]. The RS-GW model instead is dual by construction to a CFT where the most relevant deformation is almost marginal, *i.e.* $|d-4| = |\epsilon| \ll 1$. Such a deformation, see eq. (4.2), determines a large hierarchy of scales, essentially like in the Coleman-Weinberg mechanism. Of course if the GW field had a negative mass such that, say $\epsilon = -2$, the CFT would have the analogue of the Higgs mass problem in the SM. The minimum of eq. (4.2) would generically be at the cut off scale: $\langle \mu \rangle \sim \mu_0$. The RS-GW model is a (maybe ad hoc) construction of a CFT without strongly relevant deformations. However it gives a fairly predictive set up, at least when the *AdS* geometry is not too curved, i.e. when the CFT has both large N and large 't Hooft coupling. So it is worth further investigations.

5. Shining and natural flavor conservation

Any attempt at solving the hierarchy problem by invoking new physics just above the weak scale runs the risk of spoiling the good features of the SM in the matter of Flavor, Baryon and Lepton quantum numbers. In the SM the only "relevant" operators that violate flavor are the Yukawa couplings. This property goes under the name of Natural Flavor Conservation (NFC). All dangerous effects are properly suppressed by combinations of quark masses and CKM mixing angles and the resulting phenomenology agrees with the data. Similarly, baryon and lepton numbers are violated at lowest order respectively by operators of dimension 6 and 5. Provided the SM cut-off is large these properties nicely explain why we have not yet detected proton decay and why the neutrini are so light.

In the flavor sector probably the most natural assumption is that NFC keeps being valid beyond the SM. For example this possibility is realized by models with gauge mediated supersymmetry breaking[25]. Composite technicolor models [26] also aim at this principle. We can do the same thing in the RSI/CFT model and see what the consequences are. By working in the CFT picture, NFC corresponds to the requirements

1. There is an approximate global $G_F = \mathrm{SU}(3)^5$ flavor symmetry. This symmetry acts in the usual way on the low energy degrees of freedom, quarks and leptons (By assuming $G_F = \mathrm{SU}(3)^5 \times \mathrm{U}(1)_B \times \mathrm{U}(1)_L$ we could also take care of baryon and lepton number, as discussed below).

2. The CFT scalar operators with non trivial G_F quantum numbers are all strongly irrelevant, say of dimension > 5. The only exception is represented by 3 multiplets of almost marginal operators $\mathcal{O}_u$, $\mathcal{O}_d$, $\mathcal{O}_e$, with the quantum numbers of the SM Yukawa couplings (i.e. in $(\bar{3}, \bar{3})$-type representations of the flavor group).

By these requirements, the CFT can be defined also in a limit where flavor is unbroken. Then by turning on at the cut-off the most general sources of flavor violation one is guaranteed that at low energy the only effects that are not strongly power suppressed are due to the sources Y_u, Y_d and Y_e of the $\mathcal{O}_u$, $\mathcal{O}_d$, $\mathcal{O}_e$. These are almost marginal and can survive (or even grow a little) over several decades of RG evolution.

By applying the AdS/CFT dictionary points 1) and 2) translate into the following requirements on the RSI model.

1. G_F is gauged in the 5d bulk. This gauge symmetry corresponds to having the same symmetry, but global, in the CFT: the 5d flavor gauge bosons are dual to the global 4d flavor currents.

2. There are 5d scalars ϕ_u, ϕ_d and ϕ_e transforming as the corresponding Yukawas. They have bulk masses $m^2_{u,d,e}$ such that $\epsilon_{u,d,e} = \sqrt{4 + m^2_{u,d,e} L^2} - 2$ is small in absolute value. Indeed it is enough that $|\epsilon_{u,d,e}|$ be smaller than the corresponding $|\epsilon|$ of the GW field. This makes sure that Yukawas do not get too depressed by running down from the Planck scale, or that they do not destabilize the hierarchy.

3. G_F is spontaneously broken to the identity by scalar fields living on the Planck brane *only*. This breaking generates at the Planck brane sources for the ϕ's so that Yukawa couplings are generated on the TeV brane by shining [27] through the 5d bulk. The origin of flavor breaking cannot be in the bulk or on the TeV brane, cause that would mean that the inner CFT dynamics breaks flavor. Instead we want flavor to be broken by external sources, so it should happen on the Planck brane only.

We should say that we are not trying to explain the structure of fermion masses and their mixing angles. In the above description the explanation for that lies within the Planck dynamics. For instance one could imagine the Planck brane to consist of several branes all separated by some distance and then apply the ideas of refs. [27, 28]. However, for the general predictions we will discuss, all we need to know is that at the Planck brane (or at some $z \ll z_1$) the b.c. on the scalars already reproduce the SM Yukawa structure

$$\phi_u(z_0) = a_u k^{3/2} Y_u \qquad \phi_d(z_0) = a_d k^{3/2} Y_d \qquad \phi_e(z_0) = a_e k^{3/2} Y_e \tag{5.1}$$

with a's of order 1. Then the scalars in the bulk behave at lowest order in Y like (ex. ϕ_u)

$$\phi_u = \left(A z^{4+\epsilon_u} + B z^{-\epsilon_u}\right) Y_u\,, \tag{5.2}$$

where A, B are numerical coefficients fixed by the boundary conditions. For instance if the condition on the TeV brane is just $\partial_z \phi = 0$ we have that A is negligible and that $B \simeq a_u k^{3/2} z_0^{\epsilon_u}$. In this case $\phi_u \propto Y_u$ is essentially constant through the bulk. (We will further discuss the TeV brane b.c. in the next section).

The reason why this set up realizes NFC is that the only sources of flavor violation that are active close to the TeV brane are the scalar profiles, i.e. the SM Yukawa couplings. Consider for instance the masses of the $\mathrm{SU}(3)^5$ KK vectors. First of all since the group is completely Higgsed there are no zero modes. Moreover the modes with mass $M \ll \mu_0$ are localized close to the TeV brane, so that they are totally insensitive to the original sources of Flavor breaking on the Planck brane. These masses are only (mildly) sensitive to the $\phi_{u,d,e} \propto Y_{u,d,e}$ profiles. For small Yukawa the masses are given by

$$m_n = \mu_1 \left(x_n + O(Y^2)\right)\,, \tag{5.3}$$

where $x_n \simeq (n - 1/4)\pi$ are the roots of $J_0(x) = 0$. Notice that there is no massless vector even for $Y \to 0$ [29], as long as G_F is totally broken on the Planck brane. For instance vector exchange will induce flavor symmetric effective operators up to corrections quadratic in the Yukawa couplings and respecting the $\mathrm{SU}(3)^5$ selection rules. Consistency with the low energy phenomenology is easily met for vector masses of order a TeV [28].[7] Similar conclusions are reached for the KK couplings to the SM

[7] Indeed the severest constraint comes from electric dipole moments. These are however easily satisfied if one assumes that CP violation is mediated to the SM only through the $\phi_{u,d,e}$ profiles.

fermions. Up to small Yukawa corrections these are $\mathrm{SU}(3)^5$ symmetric and given by

$$g_i^2 = \frac{\bar{g}_i^2}{L} \qquad i = 1, \ldots, 5\,, \tag{5.4}$$

where $\bar{g}$ are the 5d couplings. We conclude that NFC leads in the RS scenario to a reach set of predictions which are in principle testable at future colliders.

Notice that each factor $\mathrm{SU}(3)_i$ in $\mathrm{SU}(3)^5$ has a cubic anomaly a_i from the SM fermions localized on the TeV brane. This localized anomaly can be taken care of by the proper Chern-Simons (CS) term in the bulk

$$L_{CS} = \frac{a_i}{16\pi^2} \int (A \wedge dA \wedge dA + \cdots)\,. \tag{5.5}$$

Then there should also be a $-a_i$ anomaly due to fermions on the Planck brane. These fermions are however of no consequence: they have mass $O(\mu_0)$ as G_F is broken on the UV brane, and moreover they couple very weakly [29, 19] to the light KK vector bosons.

If we want to explain baryon and lepton number conservation we should also gauge them. However both $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_L$ have mixed SM anomalies like for instance $\mathrm{U}(1)_B\,\mathrm{SU}(2)_W^2$ which cannot be eliminated by a CS term. This is because the SM gauge group is localized on the brane. For B and L we are forced to add extra matter charged under $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ on the TeV brane to cancel the mixed anomalies [1]. The B^3, BL^2, B^2L and L^3 can still be cured by the CS term. With these proviso we may then imagine that B and L are broken on the Planck brane. Now, to be consistent with proton decay data, we will have to assume that the lightest bulk scalar with $B = 1$ and $L = -1$ has a mass $m^2L^2 > 12$. On the other hand in order to naturally obtain the neutrino mass observed at SuperKamiokande we need a bulk scalar with $B = 0$, $L = 2$ and mass $m^2L^2 \sim 4 \div 5$.

6. Radius stabilization by electroweak breaking

In this section we want to consider some variations over the minimal GW set up. So far we have assumed that the boundary potentials $V_0(\phi)$ and $V_1(\phi)$ select some fixed values $\tilde{v}_0$ and $\tilde{v}_1$ for ϕ. This means that we considered the limit where $a_2, b_2 \to \infty$ in the expansion

$$\begin{aligned} V_0 &= \frac{a_2}{2}(\phi - \tilde{v}_0)^2 + \frac{a_3}{3}(\phi - \tilde{v}_0)^3 + \cdots \\ V_1 &= \frac{b_2}{2}(\phi - \tilde{v}_1)^2 + \frac{b_3}{3}(\phi - \tilde{v}_1)^3 + \cdots. \end{aligned} \tag{6.1}$$

As discussed already in ref. [12] (see also [16]) the GW mechanism can work also in the more general case. As a different, and interesting, parameter choice let us take b_2 finite and $\tilde{v}_1 = 0$. The latter choice would be the right one for the flavor fields of the previous section, as we assumed that flavor is not broken by the CFT inner dynamics. To simplify things we also assume a small source in order to neglect cubic and higher terms in the ϕ action. The boundary condition at the TeV brane is

$$-2z_1\partial_z\phi = L\partial_\phi V_1(\phi) \simeq b_2 L\phi \tag{6.2}$$

while at the Planck brane it is still $\phi(z_0) = \tilde{v}_0$. The solution is then given by

$$\phi(z) = \tilde{v}_0(z_0/z)^\epsilon \frac{1-(z/z_1)^{4+2\epsilon}\eta}{1-(z_0/z_1)^{4+2\epsilon}\eta} \qquad \eta = \frac{b_2L-2\epsilon}{8+2\epsilon+b_2L} \tag{6.3}$$

and the (renormalized) effective potential

$$V = \eta(4+2\epsilon)v_0^2\mu_0^{-2\epsilon}\mu^{4+2\epsilon}\,. \tag{6.4}$$

Corresponding to $\tilde{v}_1 = 0$ the VEV $\langle\mathcal{O}\rangle \propto A = -\eta v_0\mu_0^{-\epsilon}\mu^{4+\epsilon}$ vanishes with the source $v_0 = \lambda(\mu_0)$ and the potential has no piece linear in $\lambda(\mu)$. (Eq. (6.4) agrees with eq. (2.9) in the limit $v_1 = 0$, $b_2 \to \infty$.)

Now, it is easy to check that by even taking finite a_2 in V_0 the μ dependence of the renormalized potential is the same. Only the coefficient in front is affected. This just corresponds to a redefinition of the source

$$v_0 \to f(v_0) = d_1v_0 + d_2v_0^2 + \cdots \tag{6.5}$$

in eq. (6.4). Again, by modifying physics on the Planck brane we simply modify the definitions of our CFT deformation parameters, but the infrared behavior (μ dependence) remains the same. Without loss of generality we can simply fix the $\phi(z_0)$.

Eq. (6.4) by the addition of a tension term $\delta V = \beta\mu^4$ can stabilize μ. For $\epsilon > 0$ one needs $\eta > 0$ and $\beta < 0$, for $\epsilon < 0$ the reverse $\eta < 0$ and $\beta > 0$. Notice that $\eta < 0$ implies $b_2 < 0$, which does not lead to an instability as long as $b_2L \lesssim 1$.

In the RS model both the hierarchy and NFC require bulk scalars with a mass $|m^2| \ll 1/L^2$. So it is natural to assume that the role of the GW field is played by the $\phi_{u,d,e}$ themselves. Then the radion potential will be mainly determined by the field ϕ_t corresponding to the top quark Yukawa coupling. This field has the "largest" profile. By flavor conservation on the TeV brane the potential V_1 in eq. (6.1) is a function of $\phi_t\phi_t^\dagger$, so it corresponds to $v_1 = 0$. At lowest order it gives a radion potential of the form in eq. (6.4). As we said, stability can be achieved by balancing this term with $\beta\mu^4$. However with the SM living on the brane there is also another, more interesting, option. The Higgs doublet H could be involved in the stabilization dynamics. In general the coefficients b_i will depend on the bilinear $HH^\dagger$, so that at quartic order in H the effective potential for Higgs and radion will be written as

$$V = y_1(\mu)\mu^2HH^\dagger + y_2(\mu)(HH^\dagger)^2 + y_3(\mu)\mu^4\,. \tag{6.6}$$

The dominant μ dependence of the y_i's is through the top flavon source $L^{3/2}\phi_t(z_0)=v_0$

$$y_1 = (y_1)_0 + (y_1)_1 v_0^2(\mu/\mu_0)^{2\epsilon} + \cdots. \tag{6.7}$$

For instance one could consider a parameter choice where the y_i start positive at μ_0 but y_1 goes negative at lower μ. By truncating eq. (6.7) to the first two terms, the choice $(y_1)_0 > 0$, $(y_1)_1 < 0$ $\epsilon < 0$ achieves the goal. For negative y_1 one can minimize in the Higgs field and find an effective potential for μ

$$V_{\text{eff}} = \left(y_3(\mu) - \frac{y_1(\mu)^2}{4y_2(\mu)}\right)\mu^4 = y_{\text{eff}}(\mu)\mu^4. \tag{6.8}$$

Then $\langle\mu\rangle$ will roughly correspond to the point (if it exists) where y_{eff} crosses zero. Notice that a stable minimum now exists even for $y_3 \equiv 0$. A similar interplay between a dilaton type field μ and the Higgs was already suggested in no-scale inspired supersymmetric models [30]–[34] where the soft terms are proportional to a modulus. In particular, in the "phenomenological" models in refs. [31, 32] the μ dependence of the effective potential couplings is just determined by RG evolution in the MSSM. This makes them more predictive than the RS model where we, instead, have to make assumptions about the behavior of the y's. However in refs. [31, 32] a much stronger assumption on the absence of quadratic divergences in the modulus potential had to be made. Here this result simply follows from the properties of the deformations of the CFT, of which μ is part.[8]

Now, having the Higgs to play a role in radius stabilization sounds nice, but leads to no predictions unless further assumptions are made. One reasonable step then is to consider the limit where μ is weakly coupled. In the AdS picture μ is part of the gravitational field and its kinetic term is

$$L_{\text{kin}} = -12(ML)^3(\partial\mu)^2 = \frac{N^2}{2}(\partial\mu)^2\,. \tag{6.9}$$

We want to consider N large with the y's fixed for which the radion decay constant $\Lambda_\mu = N\langle\mu\rangle$ is somewhat larger that the weak scale, and the Higgs-radion mixing suppressed. At the minimum of eq. (6.8) we find (the dots represent $\mu d/d\mu$ derivatives)

$$m_\mu^2\Lambda_\mu^2 = (-16y_{\text{eff}} + \ddot{y}_{\text{eff}})\mu^4 = 2m_h^2 v_F^2\left(1 - \frac{4y_2y_3}{y_1^2}\right)(1 + O(\epsilon^2)/y_{\text{eff}}) \leq 2m_h^2 v_F^2\,, \tag{6.10}$$

where $v_F = \sqrt{2}\langle H\rangle = 246\,\text{GeV}$. The inequality is obtained neglecting terms that are generically $O(\epsilon)$ and by considering that at the minimum $y_{\text{eff}} < 0$ with $y_3 > 0$

[8] The models [30, 33, 34] are truly 5 dimensional above the weak scale: this fact and supersymmetry insure that the modulus is not badly destabilized. In order to mantain a hierarchy between the TeV and Planck scales, as it happens in the RS model, the 5d theory itself should flow to a fixed point above a TeV [34]. It would be interesting to find explict examples where this happens and for which the modulus dynamics remains calculable.

$(0 < 1 - 4y_2y_3/y_1^2 < 1)$. In the limit we considered $m_\mu \sim v_F^2/\Lambda_\mu$, like any modulus. For Λ_μ of order a few TeV we could reasonably expect a radion with a mass of a few GeV or even less. It may be worth to devote more attention to this region of parameter space, which has been so far neglected.

7. Conclusions

In this paper we have discussed the four dimensional interpretation of the compact Randall-Sundrum model. Basically we have shown that the system of Kaluza-Klein states plus the massless radion corresponds to 4d gravity coupled to a field theory with a spontaneously broken conformal invariance. This conclusion is easily reached by taking the limit where the Planck brane is at the boundary of AdS, in which 4d gravity decouples. In this limit, the reaction of the system to an external 4d metric $\bar{g}_{\mu\nu}$ can be studied by the standard AdS/CFT technique. The generating functional $S[\bar{g}]$ so obtained is invariant (up to the anomaly) under Weyl transformations regardless of the presence of the IR brane, indicating that the theory satisfies the Ward identities for conformal invariance. However the presence of the IR brane does not allow to define conformal transformations globally: a mass gap is generated and the radion is the Goldstone boson of dilatations. This result is valid at tree level in the 5d theory. It should however remain true by including quantum corrections in an effective field theory approach (like the one used for the pion lagrangian) regardless of whether there is or there isn't a fundamental string theory description of the model. This is just because of the spacetime symmetry properties of AdS space [14]. On the other hand, a string theory description would allow a complete description of the CFT, including operators of arbitrarily high dimension ($\gtrsim ML$) and spin. Correspondingly it would allow to study interactions in the energy regime $E \sim (ML)\mu_1$ where the effective lagrangian approach breaks down.

The Golberger-Wise stabilization mechanism corresponds to breaking conformal invariance explicitly by turning on a coupling λ_{GW} of dimension $-\epsilon$. This leads to a non trivial effective potential for the radion $V(\mu) = \mu^4 P(\lambda_{GW}\mu^\epsilon)$, which generically has a minimum at

$$m_{\rm weak} = \langle\mu\rangle \sim \lambda_{GW}^{-1/\epsilon}\,. \tag{7.1}$$

For $|\epsilon| \ll 1$, a natural hierarchy is generated. As λ_{GW} is an essentially dimensionless parameter, one could say that the hierarchy is generated by dimensional transmutation, like in the Coleman-Weinberg mechanism or in technicolor. But unlike technicolor the coupling λ_{GW} responsible for the hierarchy can be either strong or weak at the TeV scale (see appendix A). Which case is realized is a matter of parameter choices in the gravitational picture. In the case of a weak λ_{GW} there is a light scalar resonance, radion, that couples to the trace of the energy momentum tensor. It is amusing that Natural Flavor Conservation suggests that the SM Yukawa couplings are almost marginal. So it is possible that the role of λ_{GW} in the RS model is played by the top Yukawa coupling.

JHEP04(2001)021

Acknowledgments

We would like to thank Nima Arkani-Hamed and Massimo Porrati for many important discussions and comments. We would also like to thank Csaba Csàki, John March-Russell, Luigi Pilo, Alex Pomarol and Alessandro Strumia for stimulating conversations. R.R. thanks the hospitality of the TH division at CERN. This work is partially supported by the EC under TMR contract HPRN-CT-2000-00148 (R.R.).

A. The radion mass

For a generic potential of GW type $V(\mu) = \mu^4 P(\mu^\epsilon)$ the radion mass is given by

$$m_{rad}^2 = \frac{1}{24M^3L^3}\mu_1^{2+\epsilon}\left[(4\epsilon+\epsilon^2)P'(\mu_1^\epsilon) + \epsilon^2\mu_1^\epsilon P''(\mu_1^\epsilon)\right], \tag{A.1}$$

where we used the stationarity condition $4P(\mu_1^\epsilon) + \epsilon\mu_1^\epsilon P'(\mu_1^\epsilon) = 0$. For small ϵ the minimum is roughly at the place where P vanishes. Generically the above equation implies $m_{rad}^2 = O(\epsilon)$, which is the analogue of $m^2 = O(\alpha/4\pi)$ in the Coleman-Weinberg mechanism. For the original GW potential of eqs. (2.9), (4.5), $P(x)$ is close to having a double zero at $x = v_1/v_0$, which leads to a further suppression $m_{rad}^2 = O(\epsilon^{3/2})$. If we modify the model by adding a contribution $\delta T_1 = \epsilon v_1^2 L^{-4}$ to the IR brane tension, eq. (4.5) becomes a perfect square

$$V_{\rm new} = (4+2\epsilon)\mu^4(v_1 - v_0(\mu/\mu_0)^\epsilon)^2\,. \tag{A.2}$$

Now $V_{\rm min} = 0$, like in a supersymmetric model, while we have exactly $\mu_1/\mu_0 = (v_1/v_0)^{1/\epsilon}$. This potential is the one we would have obtained in the model of ref. [35], by working at leading order in $v^2/(ML)^3$. The advantage of that model is that one can also easily find the exact solution of the equations of motions including the backreaction [16]. So it is an ideal model to verify the consistency of the effective lagrangian approach to the radion potential. By eqs. (A.1), (A.2) we have

$$m_{\rm rad}^2 = \frac{v_1^2}{6M^3L^3}(2+\epsilon)\epsilon^2\mu_1^2 \tag{A.3}$$

which agrees with [35, eq. (6.6)].

It is interesting to consider the radion mass from the broader perspective of a 4d CFT. As discussed above the slow evolution $d\ln\lambda/d\ln\mu = \epsilon$ leads to a radion that is somewhat lighter than the other modes. In general, however, the full β-function will be

$$\frac{d\ln\lambda}{d\ln\mu} = \epsilon + b_1\lambda + b_2\lambda^2 + \cdots = \beta(\lambda) \tag{A.4}$$

and for large enough λ the evolution may become non perturbative and fast. In order to generate a hierarchy it is only necessary that λ starts running slowly at the cut-off scale μ_0. Then, if λ is irrelevant ($\epsilon > 0$) it will keep its slow run until the point where $V(\mu) = \mu^4 P(\lambda(\mu))$ is minimized, and the radion mass will be suppressed. However for the more interesting case of a relevant ($\epsilon < 0$) or marginally relevant ($\epsilon = 0$, $b_1 < 0$ for λ positive) coupling the size of $m_{\rm rad}$ depends on whether $\lambda(\mu)$ is already running fast at the minimum. If all the a_n and b_n in the expansion of respectively P (see eq. (4.2)) and $\beta(\lambda)$ were of order 1 (apart from $b_0 = \epsilon$), then also $\lambda(\mu) = O(1)$ at the minimum and the radion mass would be unsuppressed. In a sense this is what happens in technicolor models. Here the deformation λ is a gauge coupling g^2 for which $\epsilon = 0$ and $b_1 < 0$, the scale μ is precisely the one where λ becomes non-perturbative and there is nothing looking like a light radion in the spectrum. On the other hand, if, for instance, $a = P(0)$ is small then $P(\lambda)$ can cross zero for a perturbative λ and the radion mass be suppressed. (This is truly what happens in the Coleman-Weinberg scalar electrodynamics example [13], where the scalar quartic coupling is playing the role of a and the gauge coupling the role of λ). This situation must probably be realized if λ coincides with the top Yukawa coupling λ_t: the fact that λ_t is perturbative in the low energy theory is evidence that it is slowly running in the full CFT just above the weak scale.

B. Two-point functions

In this appendix, we compute the two-point function of a minimally coupled scalar field in the RSI model, corresponding to a dimension four operator O of the CFT. The same computation also gives the transverse-traceless part of the two-point function of the stress-energy tensor. We closely follow the analogous computation in AdS/CFT [14]. Put the UV brane at $z = R$ (as a UV regulator) and the TeV brane at $z = 1/\mu$. We are interested in the limit where 4d gravity is decoupled and the UV brane is sent to the boundary $z = 0$, therefore at the end we will take the limit $R \to 0$. Consider the Fourier component ϕ_p of a minimally coupled massless scalar field. In the coordinates of eq. (2.2), its equation of motion reads $(pz)^2\phi'' - 3(pz)\phi' - (pz)^2\phi = 0$, whose solution is

$$\phi_p(z) = (pz)^2 \left[A(p)K_2(pz) + B(p)I_2(pz)\right]. \tag{B.1}$$

In the ordinary AdS/CFT computation [14], regularity at the horizon $z = \infty$ selects the solution with $B(p) = 0$. We choose a Neumann condition $\partial_z\phi(z = 1/\mu) = 0$ at the TeV brane, which fixes $B(p) = A(p)K_1(p/\mu)/I_1(p/\mu)$. The value $\phi^0_p = \phi_p(R)$ is identified with the boundary source for the conformal operator O. The generating functional $W[\phi^0]$ for the connected Green functions for O can be computed from supergravity as the bulk action evaluated on the solution (B.1) [14]. As usual, the latter reduces to a boundary term

$$\langle O(p)O(-p)\rangle = \frac{\partial^2 W}{\partial\phi^0_p\partial\phi^0_{-p}} = \left[\frac{1}{z^4}\frac{\phi_p(z)z\partial_z\phi_p(z)}{\phi_p(R)^2}\right]^{z=R}_{z=1/\mu}. \tag{B.2}$$

The contribution at the TeV brane is identically zero due to the boundary condition. Taking $R \to 0$, the computation simplifies

$$\langle O(p)O(-p)\rangle = \left[\frac{1}{R^4} z\partial_z \log\phi\right]^{z=R} = -\frac{p^4}{4}\left(\log\frac{pR}{2} - \frac{K_1(p/\mu)}{I_1(p/\mu)}\right) . \tag{B.3}$$

The two-point function reduces to the conformal result $-(k^4\log k)/4$ in the UV limit $k \gg \mu$, with exponential corrections, and it is analytic for $k \ll \mu$, where the log is exactly canceled by the Bessel functions.

The transverse-traceless part of the stress-energy tensor two-point function as defined in eq. (3.10) can be computed in terms of the previous expression,

$$\mathcal{F}(p) = \int d^4p e^{ipx}\frac{c(x)}{x^4} = 8\pi^2 c\frac{\langle O(p)O(-p)\rangle}{p^4} , \tag{B.4}$$

where the normalization factor has been introduced to match the UV conformal result $\mathcal{F}(p) \sim -2\pi^2 c\log p$ [14].

References

[1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *The hierarchy problem and new dimensions at a millimeter*, *Phys. Lett.* **B 429** (1998) 263 [hep-ph/9803315]; *Phenomenology, astrophysics and cosmology of theories with sub-millimeter dimensions and TeV scale quantum gravity*, *Phys. Rev.* **D 59** (1999) 086004 [hep-ph/9807344].

[2] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *New dimensions at a millimeter to a fermi and superstrings at a TeV*, *Phys. Lett.* **B 436** (1998) 257 [hep-ph/9804398].

[3] For previous relevant literature see also: I. Antoniadis, *A possible new dimension at a few TeV*, *Phys. Lett.* **B 246** (1990) 377;
E. Witten, *Strong coupling expansion of Calabi-Yau compactification*, *Nucl. Phys.* **B 471** (1996) 135 [hep-th/9602070];
P. Hořava and E. Witten, *Heterotic and type-I string dynamics from eleven dimensions*, *Nucl. Phys.* **B 460** (1996) 506 [hep-th/9510209]; *Eleven-dimensional supergravity on a manifold with boundary*, *Nucl. Phys.* **B 475** (1996) 94 [hep-th/9603142];
J.D. Lykken, *Weak scale superstrings*, *Phys. Rev.* **D 54** (1996) 3693 [hep-th/9603133].

[4] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].

[5] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[6] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

JHEP04(2001)021

[7] E. Witten, talk at ITP conference *New dimensions in field theory and string theory*, Santa Barbara `http://www.itp.ucsb.edu/online/susy_c99/discussion`.

[8] S.S. Gubser, *AdS/CFT and gravity*, *Phys. Rev.* **D 63** (2001) 084017 [`hep-th/9912001`].

[9] H. Verlinde, *Holography and compactification*, *Nucl. Phys.* **B 580** (2000) 264 [`hep-th/9906182`].

[10] H. Verlinde, *Supersymmetry at large distance scales*, `hep-th/0004003`.

[11] N. Arkani-Hamed, M. Porrati and L. Randall, *Holography and phenomenology*, `hep-th/0012148`.

[12] W.D. Goldberger and M.B. Wise, *Modulus stabilization with bulk fields*, *Phys. Rev. Lett.* **83** (1999) 4922 [`hep-ph/9907447`].

[13] S. Coleman and E. Weinberg, *Radiative corrections as the origin of spontaneous symmetry breaking*, *Phys. Rev.* **D 7** (1973) 1888.

[14] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [`hep-th/9802109`];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [`hep-th/9802150`].

[15] M. Henningson and K. Skenderis, *The holographic Weyl anomaly*, *J. High Energy Phys.* **07** (1998) 023 [`hep-th/9806087`];
Holography and the Weyl anomaly, *Fortsch. Phys.* **48** (2000) 125 [`hep-th/9812032`].

[16] O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch, *Modeling the fifth dimension with scalars and gravity*, *Phys. Rev.* **D 62** (2000) 046008 [`hep-th/9909134`].

[17] C. Csáki, M. Graesser, L. Randall and J. Terning, *Cosmology of brane models with radion stabilization*, *Phys. Rev.* **D 62** (2000) 045015 [`hep-ph/9911406`].

[18] W.D. Goldberger and M.B. Wise, *Phenomenology of a stabilized modulus*, *Phys. Lett.* **B 475** (2000) 275 [`hep-ph/9911457`].

[19] A. Pomarol, *Grand unified theories without the desert*, *Phys. Rev. Lett.* **85** (2000) 4004 [`hep-ph/0005293`].

[20] N. Kaloper, E. Silverstein and L. Susskind, *Gauge symmetry and localized gravity in M-theory*, `hep-th/0006192`.

[21] D. Anselmi, *Central functions and their physical implications*, *J. High Energy Phys.* **05** (1998) 005 [`hep-th/9702056`].

[22] I.R. Klebanov and E. Witten, *AdS/CFT correspondence and symmetry breaking*, *Nucl. Phys.* **B 556** (1999) 89 [`hep-th/9905104`].

[23] P.H. Frampton and C. Vafa, *Conformal approach to particle phenomenology*, hep-th/9903226.

[24] C. Csáki, W. Skiba and J. Terning, *Beta functions of orbifold theories and the hierarchy problem*, *Phys. Rev.* **D 61** (2000) 025019 [hep-th/9906057].

[25] M. Dine, A.E. Nelson, Y. Nir and Y. Shirman, *New tools for low-energy dynamical supersymmetry breaking*, *Phys. Rev.* **D 53** (1996) 2658 [hep-ph/9507378];
for a review see G.F. Giudice and R. Rattazzi, *Theories with gauge-mediated supersymmetry breaking*, *Phys. Rep.* **322** (1999) 419 [hep-ph/9801271].

[26] H. Georgi, *A tool kit for builders of composite models*, *Nucl. Phys.* **B 266** (1986) 274.

[27] N. Arkani-Hamed and S. Dimopoulos, *New origin for approximate symmetries from distant breaking in extra dimensions*, hep-ph/9811353.

[28] N. Arkani-Hamed, L. Hall, D. Smith and N. Weiner, *Flavor at the TeV scale with extra dimensions*, *Phys. Rev.* **D 61** (2000) 116003 [hep-ph/9909326].

[29] A. Pomarol, *Gauge bosons in a five-dimensional theory with localized gravity*, *Phys. Lett.* **B 486** (2000) 153 [hep-ph/9911294].

[30] I. Antoniadis, C. Muñoz and M. Quirós, *Dynamical supersymmetry breaking with a large internal dimension*, *Nucl. Phys.* **B 397** (1993) 515 [hep-ph/9211309].

[31] C. Kounnas, F. Zwirner and I. Pavel, *Towards a dynamical determination of parameters in the minimal supersymmetric standard model*, *Phys. Lett.* **B 335** (1994) 403 [hep-ph/9406256].

[32] R. Barbieri and A. Strumia, *Relating $m_{\rm Higgs}$ to $m_{\rm SUSY}$ by a loop factor*, *Phys. Lett.* **B 490** (2000) 247 [hep-ph/0005203].

[33] I. Antoniadis, S. Dimopoulos and G. Dvali, *Millimeter range forces in superstring theories with weak-scale compactification*, *Nucl. Phys.* **B 516** (1998) 70 [hep-ph/9710204].

[34] I. Antoniadis, S. Dimopoulos, A. Pomarol and M. Quiros, *Soft masses in theories with supersymmetry breaking by TeV-compactification*, *Nucl. Phys.* **B 544** (1999) 503 [hep-ph/9810410].

[35] C. Csáki, M.L. Graesser and G.D. Kribs, *Radion dynamics and electroweak physics*, *Phys. Rev.* **D 63** (2001) 065002 [hep-th/0008151].